NORTH SYDNEY BOYS HIGH SCHOOL

2011 HSC ASSESSMENT TASK 4

Mathematics

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

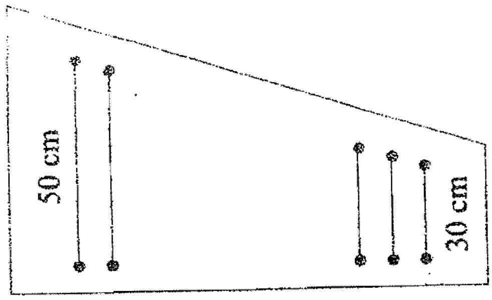
- O Mr Berry
- O Ms Ziaziaris
- O Mr Lin
- O Mr Weiss
- O Mr Lam

Student Number:__________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	Total	Total %
Mark	/10	/5	/6	/6	/4	/8	/9	/48	/100

1.a)



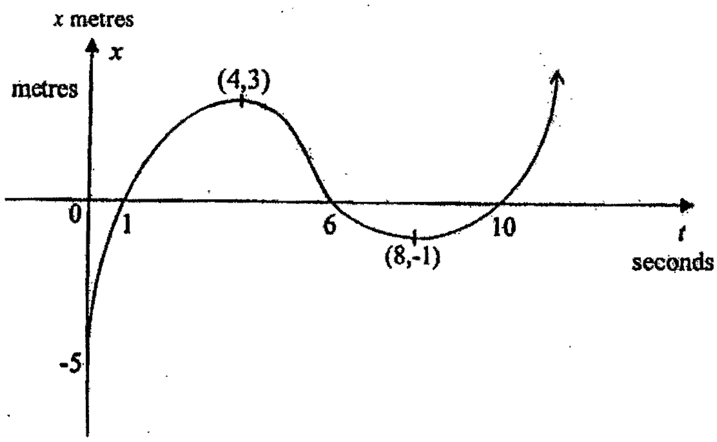
NOT TO SCALE

A simple instrument has many strings, attached as shown in the diagram. The difference between the lengths of adjacent strings is a constant, so that the lengths of the strings are the terms of an arithmetic series.

The shortest string is 30 cm long and the longest string is 50 cm. The sum of the lengths of all the strings is 1240 cm.

(i)	Find the number of strings.	2
(ii)	Find the difference in length between adjacent strings.	2

b)



NOT TO SCALE

The graph shows the displacement, x metres from the origin, at any time t seconds, of a particle moving in a straight line.

i)	Where was the particle initially?	1
ii)	When was the particle at the origin?	1
iii)	When was the particle at rest?	2
iv)	Estimate the time when the acceleration was zero?	1
v)	Find the total distance travelled by the particle during the first 10 seconds?	1

2. An experimental vaccine was injected into a cat. The amount, M millilitres, of vaccine present in the bloodstream of the cat, t hours later was given by M = $e^{-2t}+3$.

i) How much vaccine was initially injected into the cat? 1

ii) At what rate was the amount of vaccine decreasing at the end of 3 hours? 2

iii) Show that there will always be more than 3 millilitres of vaccine present in the cat's bloodstream. 1

iv) Sketch the curve of M = $e^{-2t}+3$ to show how the amount of vaccine present in the cat's bloodstream changes over time. 1

3. A particle moves in a straight line. At time t seconds, its displacement, x metres from a fixed point O on the line is given by

$$x = 1 - \cos \pi t$$

i) What is the initial displacement of the particle? 1

ii) Sketch the graph as a function of t for $0 \le t \le 2$. 2

iii) Find an expression for the velocity of the particle at any time t. 1

iv)What is the velocity of the particle at time $t = \frac{1}{6}$ 1

v)At what time does the particle first reach its maximum speed? 1

4. Initially a population contains 1000 individuals. After t years the number N of individuals in the population is such that $N = N_o e^{-kt}$.

i) Find the value of N_o. 1

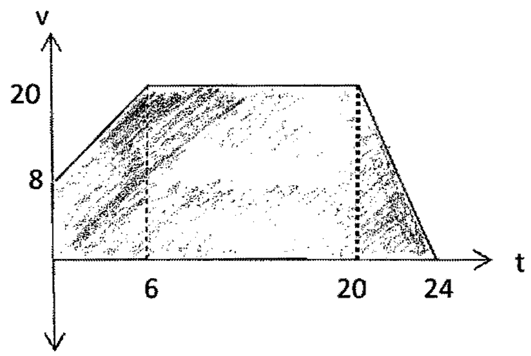
ii) Find the value of k as an exact value given that the population is 880 after 5 years. 2

iii) What is the population after 10 years? (Answer to the nearest ten people) 1

iv)Find the rate at which the population is decreasing when it is half its original size. 2

(Give answer to 2 decimal places)

5.



NOT TO SCALE

Using the velocity- time graph:

i)Find the average acceleration during the first 6 seconds. 1

ii) Explain what is happening with velocity and acceleration between t=6 and t=20 2

iii) What does the shaded area in the graph represent. Answer in one sentence. 1

6. A particle is moving in a straight line. It starts from the origin and at time t seconds its velocity v ms^{-1} is given by $v = 3t^2 + 6t - 9$.

i)	Find the initial speed of the particle and its initial direction of motion.	2
ii)	Find when and where the particle is at rest.	4
iii)	Find the position of the particle after 2 seconds.	1
iv)	Find the distance travelled by the particle in the first 2 seconds.	1

7. On 1 July 2001, Carly invested \$10 000 in a bank account that paid interest at a fixed rate of 12% per annum, compounded annually.

   (i) How much would be in the account after the payment of interest on 1 July 2011 if no additional deposits were made? **2**

   (ii) In fact, Carly added \$1000 to her account on 1 July each year, beginning on 1 July 2002.
   How much was in her account on 1 July 2011 after the payment of interest and her deposit? **4**

   (iii) Carly's friend, Minh, invested \$10 000 in an account at another bank on 1 July 2001 and made no further deposits. On 1 July 2011, the balance of Minh's account was \$40 445.

   What was the annual rate of compound interest paid on Minh's account. (Answer to the nearest whole number) **3**

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

SOLUTIONS – 2UNIT ASSESS TSK 4 YR12

① a) $30 + \ldots\ldots + 50$

(i) $S_n = \frac{n}{2}(a + l)$

$1240 = \frac{n}{2}(30 + 50)$ — 1

$1240 = 40n$

$n = 31$ — 1

31 strings

(ii) $T_{31} = 50$

$50 = a + (n-1)d$ — 1

$50 = 30 + 30d$

$20 = 30d$

$d = \frac{2}{3}$ — 1

$\therefore \frac{2}{3}$cm difference in length between strings.

b) (i) $x = -5$ OR 5m to the left. — 1

(ii) $t = 1, 6, 10$ s. — all 3 for 1

(iii) $t = 4, 8$ s — 2

(iv) Anything between $t = 5$ and $t = 5\frac{1}{2}$ — 1

(v) Distance $= 5 + 3 + 3 + 1 + 1$

$= 13$m. — 1

② $M = e^{-2t} + 3$

(i) $t = 0$

$M = e^{0} + 3$

$M = 4$mL. — 1

(ii) $\frac{dM}{dt} = -2e^{-2t}$ — 1

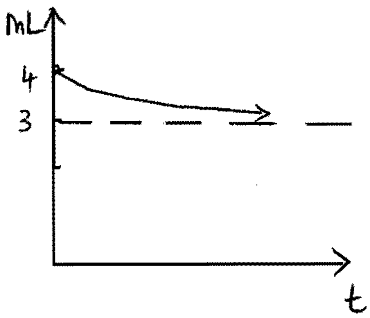
When $t = 3$

$\frac{dM}{dt} = -2e^{-6}$ — 1

$= \frac{-2}{e^{6}}$ mL/h OR -4.96×10^{-3} mL/h (2 dp)

(iii) See below

(iv) — 1



(iii) As $t \to \infty$

$e^{-2t} \to 0$.

$\therefore M \to 0 + 3 = 3$ — 1

$\therefore$ There will always be 3mL of vaccine in the cat.

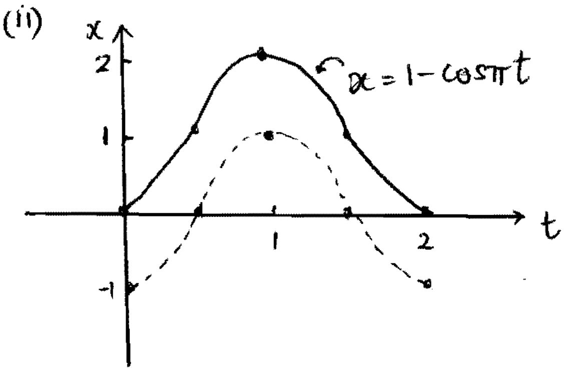
③ $x = 1 - \cos \pi t$

(i) $t = 0$

$x = 1 - \cos 0$

$x = 0$ ie. Particle is at the origin — 1

(ii)



1 for period

1 for shape

$T = \frac{2\pi}{\pi} = 2$

(iii) $v = \pi \sin \pi t$ — 1

(iv) $t = \frac{1}{6}$

$v = \pi \sin \frac{\pi}{6}$

$v = \frac{\pi}{2}$ m/s. — 1

(v) From graph, $t = \frac{1}{2}$ (ie. at the point of inflexion, $a = 0$) — 1

④ $N = N_0 e^{-kt}$

(i) $N_0 = 1000$ — 1

(ii) $880 = 1000 e^{-5k}$

$\frac{880}{1000} = e^{-5k}$ — 1

$\ln\left(\frac{880}{1000}\right) = -5k$

$k = -\frac{1}{5}\ln\left(\frac{880}{1000}\right)$ or $-\frac{1}{5}\ln\left(\frac{22}{25}\right)$ — 1

(iii) When $t = 10$

$N = 1000 e^{-10k}$

$N = 774.4 = 770$ to nearest ten — 1

(iv) $N = 500$

$\frac{dN}{dt} = -kN$ — 1

$= -500k$

$= -12.78$ individuals/year — 1

⑤ (i) Av. Acceleration $= \frac{20-8}{6-0}$

$= 2\ m/s^2$ OR $2\ units/s^2$ — 1

(ii) Velocity is constant at 20 m/s (or 20 u/s) — 1

Acceleration is zero. — 1

(iii) Distance travelled by particle in first 24 seconds. — 1

⑥ $v = 3t^2 + 6t - 9$

(i) $t = 0$

$v = -9\ m/s$.

Speed is 9 m/s and moving left. — 2

(ii) At rest: $v = 0$ — 1

$0 = 3t^2 + 6t - 9$

$0 = t^2 + 2t - 3$

$0 = (t+3)(t-1)$

$t = -3,\ t = 1$ but $t > 0$.

$\therefore t = 1$ only sol'n — 1

$x = \frac{3t^3}{3} + \frac{6t^2}{2} - 9t + C$

$t = 0,\ x = 0\ \therefore C = 0$

$\therefore x = t^3 + 3t^2 - 9t$. — 1

At $t = 1$

$x = 1 + 3 - 9$

$x = -5\ m$. — 1

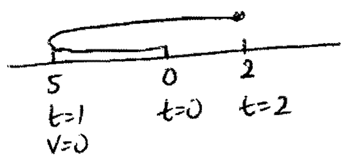
At rest after 1 second, 5m to the left of the origin.

(iii) When $t = 2$

$x = 2^3 + 12 - 18$

$x = 2m$. — 1

(iv)



$\therefore$ Distance $= 5 + 5 + 2$

$= 12m$. — 1

Q. (i) $A_n = P(1+R)^n$ 1

$A_{10} = 10\,000(1.12)^{10}$

$A = \$31058.48$ 1

(ii) The above amount plus,

1st \$1000 will be invested for 9 years & will amt to 1000×1.12^9

2nd \$1000 will be invested for 8 years & will amt to 1000×1.12^8 1

and so on until

the last \$1000 will be invested for 1 year & will amt to $\underline{1000}$

Value in Account $= 31058.48 + [1000 + 1000 \times 1.12 + 1000 \cdot 1.12^2 + \ldots + 1000(1.12)^9]$

$= 31058.48 + 1000(1 + 1.12 + 1.12^2 + \cdots + 1.12^9)$ 1

The parentheses form a GS

$S_n = \dfrac{a(r^n - 1)}{r-1}$

$= \dfrac{(1.12^{10} - 1)}{0.12}$ 1

$\therefore$ Value in account $= 31058.48 + 1000\left(\dfrac{(1.12^{10}-1)}{0.12}\right)$

$= \$48607.22$ 1

(iii) $40445 = 10\,000(1+R)^{10}$ 1

$\dfrac{40445}{10000} = (1+R)^{10}$

$\sqrt[10]{\dfrac{40445}{10000}} = 1+R$ 1

$R = \sqrt[10]{\dfrac{40445}{10000}} - 1$

$R = 15\%$ p.a. 1